Imperfect competition, emissions tax and the Porter hypothesis

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Abstract

This paper investigates the conditions under which the design of an emissions tax can align social and private interests. Our contribution is to determine the general conditions for firms' profits and social welfare to be higher under the implementation of an emissions tax than under no tax. We consider n firms producing an homogeneous product and competing over supply schedules, which covers a continuum of imperfect competition equilibria from Bertrand to Cournot. We show that, as competition intensifies, the pass-through of the tax to consumers increases, to a point where the price rises more than offsets the net result of the investment outlay. Our analysis provides new insights into the trade-off between environmental policy, market competition and the so-called "win-win" outcome for firms and society.

Keywords: Technology; R&D; Environment; Policy; Emission tax; Subsidy; Porter Hypothesis

JEL Classification: H23; O32; O38; Q55; Q58

1 Introduction

An environmental policy such as an emissions tax is usually socially desirable as it corrects an externality. However, an environmental tax may have a negative impact on firms' profitability, which can create a perverse political economy, for example through lobbying of legislators. The private economic burden of an environmental tax may result in a social welfare enhancing tax not being adopted. This paper investigates the conditions under which the design of an emissions tax aligns social and business interests. That is, we characterise the market conditions under which an emissions tax leads to an increase in firms' profits and social welfare. Our results can be seen as determining conditions under which the strong version of the Porter hypothesis (Porter, 1991, Porter and van der Linde, 1995) holds. [1]

We consider the impact of an emissions tax in the context of an oligopolistic industry where n symmetric firms produce a homogeneous product. Production by this industry entails environmentally damaging emissions. Firms respond to the emissions tax by investing in emissions abatement R&D/technology and/or by decreasing their output.

Each firm decides on its emissions reduction investment, which deterministically results in a level of emissions abatement. The firm then chooses a supply schedule, specifying how much it will produce at various prices. This approach allows us to parameterise the intensity of competition, which varies continuously from Cournot to Bertrand strategic behaviour. (See Menezes and Quiggin, 2012, 2020).

Firms undertake end-of-pipe type of emissions abatement, where gross emissions are subsequently decreased by implementing add-on measures, for example, by using a filter or scrubber technology. (See Flagan and Seinfeld, 1988, Hocking, 1998, Bellas, 1998). The firms' investment in emissions abatement technology does not lead to gains in productivity (or attracts a "green" premium in price). Instead, in our model, the mechanism through which profits increase is the reduction in product market competition.[2]

In particular, we show that firms are more likely to benefit from an emissions tax when competition is more intense; firms have more to gain from a reduction in competition when it is more intense, and under some conditions these gains outweigh the costs imposed by the emissions tax. This necessary condition is described in terms of the emissions tax cost pass-through (the proportion of tax increase that is reflected in consumer prices). For an emissions tax to be profitable, the cost-past through must be greater than the net emissions per unit of production, adjusted for the number of firms.

We then provide an example where we determine sufficient conditions for an emissions tax to be both welfare enhancing and to increase firms' profits. For firms to benefit from an optimal tax, the emissions abatement investment must be sufficiently efficient and under a given threshold, determined by the intensity

[1] See also Ambec, Cohen, Elgie & Lanoie (2013).

[2] Heyes (2009) surveys theoretical and empirical research showing the potential for environmental regulation to reduce product market competition.

of competition and the number of firms. In the case of a Cournot duopoly or a monopoly, we conclude that firms never benefits from an environmental tax. More intense market competition increases the gains from an emissions tax as it leads to a higher cost-pass through. This example complements the analysis of Delbono and Lambertini (2022) who show that, with linear demand and quadratic costs, and when two firms producing a homogenous product compete in prices, win-win solutions arise for almost any level of environmental damage in any Bertrand-Nash equilibrium. Our example shows that win-win solutions can arise even when competition is less intense than in a Bertrand equilibrium.

This paper is organised as follows. Section 2 describes the building blocks of the model, including product demand, emissions abatement technology and competition in supply schedules. In Section 3, we analyse how an emissions tax affects the profitability of firms. In Section 4, we establish the sufficient condition for an emissions tax to benefit firms. We also characterise the optimal (second best) emissions tax and the conditions under which private and social interests are aligned, i.e., the conditions under which a benevolent regulator would choose an optimal emissions tax that also benefits the firm. Section 5 presents our example and section 6 concludes.

2 The Model

Consumers preferences for a homogenous goods are represented by the inverse demand function $P(q_i, \mathbf{q}_{-i})$ with $P(0, \mathbf{0}) = P_0 > 0, P'(\cdot) < 0, P''(\cdot) > 0$ for $q_i \geqslant 0$, where q_i denotes the amount produced by firm i, $i = 1, ..., n$. Firms face zero production costs, but the production of q_i units entails emitting $E(q_i) = e_0 q_i$ units of pollution. Without loss of generality, we set $e_0 = 1$.

If firm i invests $C(x_i)$ in emissions abatement, then i reduces its total emissions q_i by $x_i > 0$, with $C(0) = 0$, $C'(\cdot) > 0$, and $C''(\cdot) \geqslant 0$. As we see below, the assumption that total emissions are reduced (instead of marginal emissions) implies that the profit function is additively separable in the two strategic variables q_i and x_i. (See Poyago-Theotoky (2007)). For simplicity, the environmental damage caused by emissions is assumed to take a linear form:

$$D = \alpha \sum_{i=1}^{n} [q_i - x_i]. \tag{1}$$

where $\alpha > 0$ is the marginal disutility of pollution.

In the absence of a tax on emissions, firms do not invest in emissions abatement and, therefore, choose how much to produce without taking into account the externalities they cause. In contrast, firms can respond to a per-unit emissions tax t by either decreasing their output, or investing in emissions abatement R&D/technology, or both. In particular, a firm only invests in emissions abatement technology if there is a non-negative return from its investment:

$$tx_i - C(x_i) \geq 0. \tag{2}$$

In this setting, a tax on emissions leads to a reduction in the quantities produced by firms and increases investment in emissions abatement. The impact of the tax on profits, however, depends on the nature of competition and on the abatement technology, as shown in the next section.

We model imperfect competition by assuming that firms compete in supply schedules.[3] By considering families of more or less elastic supply schedules, it is possible to generate spaces of oligopoly games of which Bertrand and Cournot are polar cases. Specifically, firms compete in supply schedules given by:

$$q_i(\beta, \theta_i) = \beta P + \theta_i. \tag{3}$$

The assumption of linearity in the supply schedule is not essential for the analysis, which relies on first-order conditions related to the slope of the supply curve. (See Kao, Menezes and Quiggin (2014)). Following Menezes and Quiggin (2012, 2020), we assume a fixed value for β, the slope of the firm's supply function, and that firms choose θ_i. This assumption allow us to focus on the unique equilibrium, and avoid the multiplicity of supply function equilibria that exists in more general settings.[4] In our model, β can be interpreted as a measure of market competitiveness, where Cournot is obtained when $\beta = 0$ and Bertrand when $\beta \to \infty$. In addition to choosing θ_i, each firm i chooses the level of emissions abatement x_i.

3 The Impact of an Emissions Tax

In this section we provide a necessary condition for an emissions tax to increase firms' profits in equilibrium. We write firm i's profit function in terms of its strategic variables θ_i, its choice of abatement x_i, and the strategic choice of other firms $\boldsymbol{\theta}_{-i}$ as:

$$\pi_i(\theta_i, \boldsymbol{\theta}_{-i}, x_i) = P(\theta_i, \boldsymbol{\theta}_{-i}) q_i(\theta_i, \boldsymbol{\theta}_{-i}) - t[q_i(\theta_i, \boldsymbol{\theta}_{-i}) - x_i] - C(x_i). \tag{4}$$

Maximisation of profits yields the following first-order conditions:

$$\frac{\partial P(\theta_i, \boldsymbol{\theta}_{-i})}{\partial \theta_i} q_i(\theta_i, \boldsymbol{\theta}_{-i}) + (P(\theta_i, \boldsymbol{\theta}_{-i}) - t) \frac{\partial q_i(\theta_i, \boldsymbol{\theta}_{-i})}{\partial \theta_i} = 0, \tag{5}$$

and

$$t - \frac{\partial C(x_i)}{\partial x_i} = 0. \tag{6}$$

From (6), we can infer that any positive emissions tax always leads firms to invest in emissions abatement. Moreover, firm i's choice of x_i does not depend

[3] See, for example, Grossman (1981); Robson (1981); Turnbull (1983); Klemperer & Meyer (1989); Grant & Quiggin (1996); Delgado and Moreno (2004); Vives (2011); Menezes & Quiggin (2012, 2020), and Menezes and Pereira (2017).

[4] In Klemperer and Meyer (1989), for example, the Bertrand outcome is the unique equilibrium only under unbounded demand uncertainty and constant marginal costs.

on the choices of other firms. Given that firms have identical abatement cost functions, in equilibrium we have $x_i = x_j = x^*, \forall i \neq j$

The decision on the choice of supply function (through the choice of θ_i), however, does depend on the choices of other firms. Our focus, therefore, is in the symmetric equilibrium, where $\theta_i = \theta_j = \theta^*$, $\forall i \neq j$, yielding the symmetric market equilibrium outcome $(P^*(n\theta^*), nq^*(\theta^*))$. The first order condition (5) can then be written as:

$$\frac{\partial P^*(n\theta^*)}{\partial \theta^*} q^*(\theta^*) + (P^*(n\theta^*) - t)(n + \beta \frac{\partial P^*(\theta^*)}{\partial \theta^*}) = 0. \tag{7}$$

The equilibrium output for each firm can be expressed as:

$$q^*(\theta^*) = -\frac{(P^*(n\theta^*) - t)(n + \beta \frac{\partial P^*(n\theta^*)}{\partial \theta^*})}{\frac{\partial P^*(n\theta^*)}{\partial \theta^*}}. \tag{8}$$

Provided that $P^*(n\theta^*) - t > 0$, and (5) is satisfied, it can be readily checked that the second-order conditions for profit maximisation are also satisfied under our assumptions — a unique, symmetric Nash equilibrium exists. Next we provide a condition which needs to be met for an emissions tax to have a positive impact on firms' profits in the symmetric equilibrium characterised by FOCs (5) and (6).

3.1 The Cost Pass-Through Condition

Omitting the arguments, denoting $P^{*'} = \frac{\partial P^*}{\partial \theta^*}$ and $P^{*''} = \frac{\partial^2 P^*}{\partial \theta^{*2}}$, we defined $F(P^*, t)$ using (7) as follows:

$$F(P^*, t) \equiv P^{*'}(\theta^* + \beta P^*) + (P^* - t)(n + \beta P^{*'}) = 0 \tag{9}$$

The emissions tax cost pass-through can be determined by applying the implicit function theorem to the above expression:

$$\frac{dP^*}{dt} = -\frac{\partial F/\partial t}{(\partial F/\partial \theta^*)/(\partial P^*/\partial \theta^*)} = \frac{P^{*'}(n + \beta P^{*'})}{P^{*'}(n + 1 + 2\beta P^{*'}) + P^{*''}[\theta^* + \beta(2P^* - t)]}. \tag{10}$$

Following the notation presented in Weyl and Fabinger (2013), the cost pass-through can be written in terms of the elasticity of marginal surplus $\epsilon_{ems} = (1 + \frac{P^{*''}Q}{P^{*'}})^{-1}$ as follows:

$$\frac{dP^*}{dt} = \frac{1}{1 + (\frac{1+\beta P^{*'}}{n+\beta P^{*'}})^2[\frac{1}{\epsilon_{ems}} - \frac{(n-1)\beta P^{*'}}{n(1+\beta P^{*'})}]}. \tag{11}$$

The elasticity of margin surplus (ϵ_{ems}) measures the curvature of the logarithm of demand. A log-convex demand always has $1/\epsilon_{ems} < 0.$; if the demand is convex, then $1/\epsilon_{ems} < 1$. Important to note that in our model, the cost pass-through exceeds 1 if $\epsilon_{ems} < 0$. That is, the partial cost pass-through condition may not hold in the presence of strongly convex demand. For the remainder of this paper we will focus on the case where there is an emissions tax partial cost pass-through. i.e. $0 < \frac{dP^*}{dt} \leqslant 1$.[5]

We can now establish a necessary condition for firms to profit to increase with the emissions tax. The condition, specified in the next proposition, is not sufficient since firms' profits must still be greater than the profit without an emissions tax. In the next section, we present an example where we find a necessary and sufficient condition for profits to increase with an emissions tax.

Proposition 1 (Cost Pass-Through Condition for Profits to increase): *For each firm's profit to increases with the emissions tax, the following condition needs to be met:*

$$\frac{dP^*}{dt} > \frac{n}{n-1}\frac{q^* - x^*}{q^* + \beta[P^* - t]}. \tag{12}$$

Proof: *A firm's profit in equilibrium can be expressed as:*

$$\pi^*(t) = P^*q^* - t[q^* - x^*] - C(x^*).$$

For each firm's equilibrium profits to increase with the emissions tax, the following condition needs to be met:

$$\frac{d\pi^*}{dt} = [\frac{\partial P^*}{\partial \theta^*}q^* + (P^* - t)\frac{\partial q^*}{\partial \theta^*}]\frac{\partial \theta^*}{\partial t} - q^* + x^* > 0. \tag{13}$$

Note that

$$\frac{\partial P^*}{\partial \theta^*} = \frac{\partial P}{\partial \theta_i} + (n-1)\frac{\partial P}{\partial \boldsymbol{\theta}_{-i}}; \frac{\partial q^*}{\partial \theta^*} = \frac{\partial q_i}{\partial \theta_i} + (n-1)\frac{\partial q_i}{\partial \boldsymbol{\theta}_{-i}}. \tag{14}$$

Thus, (13) can be rewritten as:

$$\frac{d\pi^*}{dt} = \{[\frac{\partial P}{\partial \theta_i} + (n-1)\frac{\partial P}{\partial \boldsymbol{\theta}_{-i}}]q^* + (P^* - t)[\frac{\partial q_i}{\partial \theta_i} + (n-1)\frac{\partial q_i}{\partial \boldsymbol{\theta}_{-i}}]\}\frac{\partial \theta^*}{\partial t} - q^* + x^* > 0. \tag{15}$$

Given the linear supply schedule (3), we have:

$$\frac{\partial q_i}{\partial \theta_i} = 1 + \beta\frac{\partial P}{\partial \theta_i}; \frac{\partial q_i}{\partial \boldsymbol{\theta}_{-i}} = \beta\frac{\partial P}{\partial \boldsymbol{\theta}_{-i}}. \tag{16}$$

Moreover, in the interior, symmetric equilibrium:

$$\frac{\partial P}{\partial \theta_i} = \frac{\partial P}{\partial \boldsymbol{\theta}_{-i}}; \frac{\partial P}{\partial \theta^*} = n\frac{\partial P}{\partial \theta_i}. \tag{17}$$

Replacing (16) and (17) into (15), yields:(5)

$$\frac{d\pi^*}{dt} = \frac{(n-1)}{n}\frac{\partial P}{\partial \theta_i}[q^* + \beta(P^* - t)]\frac{\partial \theta^*}{\partial t} - q^* + x^* > 0.$$

This implies that (12) is required for $\frac{d\pi^*}{dt} > 0$.□

[5] See Weyl and Fabinger (2013) for a comprehensive discussion of this issue.

Condition (12) requires that the emissions tax cost pass-through must be greater than the net emissions per unit of production, adjusted by the number of firms and the intensity of competition. Note that $\frac{n}{n-1}$ is equal to 2 under duopoly, tends to 1 as $n \to \infty$, and more generally decreases with n. That is, firms are more likely to find an environmental tax profitable as the number of firms increase. It is easy from (12) that β and n are substitutes[6] in the sense that the greater the β, the more likely the necessary condition for an increase in emissions tax to be profitable is met.

The key insight is that an emissions tax facilitates the exercise of market power, and the benefit for the firms is higher when the intensity of market competition is higher, or when the number of firms is larger. For example, while not formally defined in our setting, we can show that the cost pass-through condition is never met under a monopoly ($n = 1$). An emissions tax would lead the monopolist to reduce its quantity beyond the profit-maximising, monopoly quantity.

Note that for a firm to benefit from an emissions tax (sufficient condition), its profit must be greater than under no emissions tax, $\pi^*(t) > \pi^*(0)$. This general condition, which depends on specific functional forms, can be expressed as:

$$P^*(t)q^*(t) - P^*(0)q^*(0) > t[q^*(t) - x^*(t)] + C(x^*(t)). \tag{18}$$

The above condition simply states that, for a firm to benefit from an emissions tax, the increase in revenue due to the tax must compensate for the net cost of the tax.

4 Environmental Tax and Welfare

As the necessary condition for profitability of an emissions tax (12) requires the price to increase (and, therefore, the quantity to decrease) by a certain amount, consumer surplus will necessarily decrease with a tax. A decrease in quantity, however, decreases emissions, and thus increases welfare. It follows that an emissions tax that increases firms' profits may not increase welfare, as the decline in consumer surplus may not be outweighed by the increase in profits and reduction in emissions.

The focus of this section is to determine when an emissions tax can increase social welfare. In particular, we consider a social welfare function (as a function of the tax) that is additively separable in consumer surplus, producer surplus, the revenue from the emission tax, and the social environmental damage, as follows:

$$W(t) = CS(nq^*(t)) + n\pi^*(q^*(t), t) + nt[q^*(t) - x^*(t)] - \alpha n[q^*(t) - x^*(t)]. \tag{19}$$

[6] Menezes and Quiggin (2012) provide a characterisation of the substitutability between n and β to determine the competitiveness of a market.

Since both equilibrium quantities and prices are linear functions of the emissions tax, social welfare is a quadratic function of the emissions tax.

Proposition 2 (Cost-Past Through Condition for Welfare to Increase): *For social welfare to increase with the emissions tax, the following condition needs to be met:*

$$\frac{dP^*(t)}{dt} < \frac{n(\alpha - t)[\frac{d\theta^*(t)}{dt} - \frac{dx^*(t)}{dt}]}{\beta[(n-1)P^*(t) + t - n\alpha] - q^*(t)}. \tag{20}$$

Proof: *Totally differentiating a firm's profit in equilibrium (omitting the arguments for simplicity) yields:*

$$d\pi^* = \frac{n-1}{n}[q^* + \beta(P^* - t)]dP^* - (q^* - x^*)dt.$$

Similarly, totally differentiating the welfare expression (19) yields:

$$dW = -nq^*dP^* + (n-1)[q^* + \beta(P^* - t)]dP^* - n(q^* - x^*)dt + n(q^* - x^*)dt + n(t - \alpha)(dq^* - dx^*),$$

$$= [-q^* + \beta(n-1)(P^* - t)]dP^* + n(t - \alpha)(dq^* - dx^*).$$

Therefore:

$$\frac{dW}{dt} = [-q^* + \beta(n-1)(P^* - t)]\frac{dP^*}{dt} + n(t - \alpha)(\frac{dq^*}{dt} - \frac{dx^*}{dt}).$$

Replacing $\frac{dq^*}{dt} = \frac{d\theta^*(t)}{dt} + \beta\frac{dP^*}{dt}$ *and rearranging the terms:*

$$\frac{dW}{dt} = \{\beta[(n-1)(P^* - t) + n(t - \alpha)] - q^*\}\frac{dP^*}{dt} + n(t - \alpha)(\frac{d\theta^*(t)}{dt} - \frac{dx^*}{dt}),$$

which results in the above condition.□

The numerator of (20) RHS is always non-positive because $\alpha \geqslant t$, $\frac{dx^*(t)}{dt} > 0$ and $\frac{d\theta^*(t)}{dt} < 0$. Given that the emission tax cost pass-through must be positive and is complete under a Bertrand setting ($0 < \frac{dP^*(t)}{dt} \leqslant 1$) the denominator of (20) RHS must also be negative: $\beta[(n-1)P^*(t) + t - n\alpha] - q^*(t) < 0$. This is always satisfied in a Cournot setting ($\beta = 0$) and in a Bertrand setting ($P^* = t = \alpha$).

4.1 Social and Private gains from an increase in Emissions Tax

The condition for the firm's profit to increase with an increase in the tax on emissions is given by *(12)*. The condition for the social welfare to increase with an increase in the tax on emissions is given by *(20)*. Combining these two conditions, and expressing it again in terms of the emissions tax cost pass-through, yields:

$$\frac{n}{n-1}\frac{q^*(t)-x^*(t)}{q^*(t)+\beta[P^*(t)-t]} < \frac{dP^*(t)}{dt} < \frac{n(\alpha-t)[\frac{d\theta^*(t)}{dt}-\frac{dx^*(t)}{dt}]}{\beta[(n-1)P^*(t)+t-n\alpha]-q^*(t)}. \quad (21)$$

This condition translates into a range of possible emissions taxes, assuming that the equilibrium profit function is convex ($\frac{d^2\pi_e}{dt^2}>0$) and the social welfare function is concave ($\frac{d^2W}{dt^2}<0$). We can then now turn to examining when we expect this to occur. We first consider the profit function:

$$\frac{d^2\pi^*}{dt^2} = \frac{d}{dt}\{\frac{n-1}{n}\frac{dP^*}{dt}[q^*+\beta(P^*-t)]-(q^*-x^*)\},$$

$$= \frac{n-1}{n}\{\frac{d^2P^*}{dt^2}[q^*+\beta(P^*-t)]+\frac{dP^*}{dt}[\frac{dq^*}{dt}+\beta(\frac{dP^*}{dt}-1)]\}-(\frac{dq^*}{dt}-\frac{dx^*}{dt})\},$$

$$= \frac{n-1}{n}\frac{dP^*}{dt}[\frac{dq^*}{dt}+\beta(\frac{dP^*}{dt}-1)]-(\frac{dq^*}{dt}-\frac{dx^*}{dt})\},$$

The above expression has to be positive to ensure convexity of the profit function. In this case, the emissions tax that minimises the profit function (t_{em}) can be obtained by solving the following equation ($\frac{d\pi^*}{dt}=0$):

$$\frac{n-1}{n}\frac{dP^*}{dt}[q^*+\beta(P^*-t)]-(q^*-x^*)=0.$$

We now consider the social welfare function:

$$\frac{d^2W}{dt^2} = \frac{d}{dt}\{[-q^*+\beta(n-1)(P^*-t)]\frac{dP^*}{dt}+n(t-\alpha)(\frac{dq^*}{dt}-\frac{dx^*}{dt})\}.$$

$$= [-\frac{dq^*}{dt}-\beta(n-1)(1-\frac{dP^*}{dt})]\frac{dP^*}{dt}+n(\frac{dq^*}{dt}-\frac{dx^*}{dt}).$$

The above expression has to be negative to ensure concavity of the welfare function. In this case, the emissions tax that maximises the welfare function (t_{opt}) can be obtained by solving the following equation ($\frac{dW}{dt}=0$):

$$[-q^*+\beta(n-1)(P^*-t)]\frac{dP^*}{dt}+n(t-\alpha)(\frac{dq^*}{dt}-\frac{dx^*}{dt})=0.$$

The range of possible emissions taxes that ensures social and private gains (t_{w-w} to denote win-win outcomes), assuming that the equilibrium profit function is convex ($\frac{d^2\pi_e}{dt^2}>0$) and the social welfare function is concave ($\frac{d^2W}{dt^2}<0$), is given by:

$$t_{em} < t_{w-w} < t_{opt}. \quad (22)$$

In the following Sections we will use specific functional forms to illustrate the conditions under which private and social benefits can occur as a result of an emissions tax.

5 An example

Following Singh and Vives (1984), the representative consumer has a quadratic (strictly concave) utility function, resulting in the following inverse demand function:

$$P = a - (q_i + \mathbf{q}_{-i}). \tag{23}$$

Without loss of generality we will assume that $a = 1$. As in d'Aspremont and Jacquemin (1988), we assume that, if firm i invests $\frac{\nu}{2}x_i^2$ in emissions abatement R&D/technology, then firm i reduces its gross emissions q_i by x_i, where x_i represents the outcome of the firm's emissions reduction effort. Finally, ν relates to the efficiency or productivity of the abatement technology, where a higher value means lower efficiency. The investment in abatement technology is now expressed as:

$$C(x_i) = \frac{\nu}{2}x_i^2. \tag{24}$$

Firm i's profit function[7] is given by:

$$\pi_i(q_i, \mathbf{q}_{-i}, x_i) = (1 - q_i - q_{-i})q_i - t(q_i - x_i) - \frac{\nu}{2}x_i^2. \tag{25}$$

The quantities can be expressed in supply schedules and the functional objective (profit function) above can also be expressed in terms of the strategic variables $\theta_i, \boldsymbol{\theta}_{-i}$ and x_i.

At the game's profit maximising equilibrium conditions we obtain:

$$\theta^* = \frac{[1 + (n-2)\beta] - t(1 + n\beta)[1 + (n-1)\beta]}{1 + n[1 + \beta(n-1)]}, \tag{26}$$

$$x^* = \frac{t}{\nu}. \tag{27}$$

The equilibrium quantities and prices can now be determined as follows:

$$q^* = \frac{(1-t)[1 + \beta(n-1)]}{1 + n[1 + \beta(n-1)]}, \tag{28}$$

[7] Our model is equivalent to that of Lahiri & Symeonidis (2007) and Fikru and Gautier (2016). In their model, the abatement cost in the profit function is expressed in terms of emissions: $\pi_i(q_i, e_i) = (P - c)q_i - \frac{(\delta q_i - e_i)^2}{2} - e_i t_e$. Denoting $x_i = \delta q_i - e_i$ the profit function can now be written as $\pi_i(q_i, e_i) = (P - c)q_i - (\delta q_i - e_i)t_e - \frac{x_i^2}{2}$, which is similar to our model. Requate (2006) uses a generalised version of these cost functions: $C = C(q_i, e_i)$.

$$P^* = \frac{nt[1+\beta(n-1)]}{1+n[1+\beta(n-1)]}. \tag{29}$$

Substituting the equilibrium variables in the profit function we obtain:

$$\pi^*(t) = \frac{(1-t)^2[1+(n-1)\beta]}{\{1+n[1+\beta(n-1)]\}^2} + \frac{t^2}{2\nu}. \tag{30}$$

Social welfare is defined by consumer surplus plus the profits of the n-firms (excluding taxes and subsidies) minus the environmental damage, as per the following expression:

$$W = CS + n\overline{\pi}_i - D = \tag{31}$$

$$= \frac{nt(2\alpha - t) + (1-t)v(1+t-2\alpha)}{2\nu} -$$

$$-\frac{(1-t)^2}{2[n+1+n\beta(n-1)]^2} + \frac{(1-t)(\alpha-t)}{n+1+n\beta(n-1)}.$$

We now proceed to illustrate when the general conditions established in the previous section for a win-win outcome from the introduction of an emissions tax will be met in our specific example.

5.1 Tax on Emissions and Firm's Profits

The equilibrium profit function is quadratic and convex in relation to the emissions tax as depicted below:

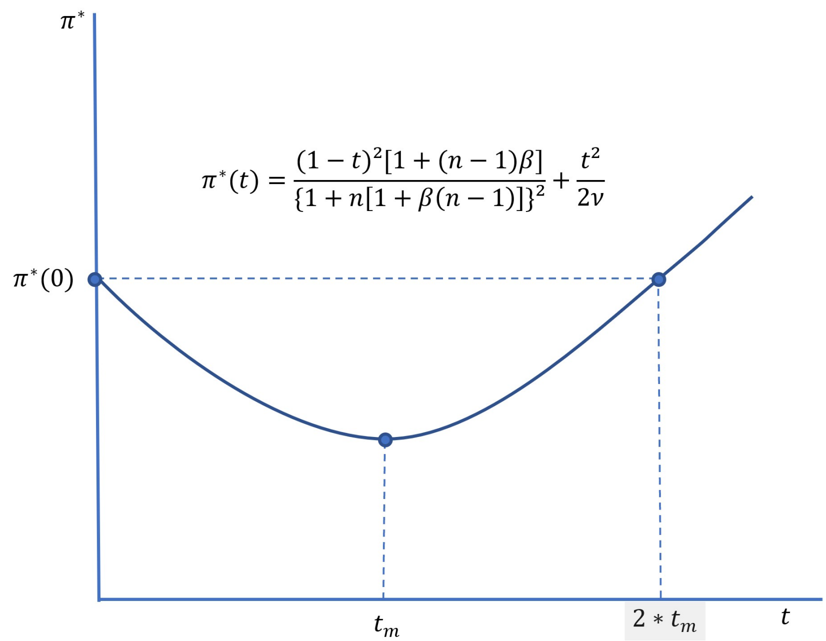

Figure 1: Profit Function

The profit function reaches its minimum at $t = t_m$:

$$t_m = \frac{2\nu[1+(n-1)\beta]}{1+n(n+2)+2\{\nu+\beta(n-1)[(n(n+1)+\nu)]\}+(n-1)^2n^2\beta^2}. \tag{32}$$

Given the quadratic and convex profit function (30), Proposition 1, which determines the cost pass-through condition for the firm's profit to increase with an increase in emissions tax, can simply be stated as:

$$t > t_m. \tag{33}$$

For the firm to benefit from an emissions tax, its profit must be greater than under no emissions tax ($\pi^*(t) > \pi^*(0)$). This condition can be stated as:

$$t\ |_{private\_benefit} > 2 * t_m. \tag{34}$$

The positive emissions condition ($q_i - x_i > 0$) must also be met, which translates into the following condition on the tax on emissions:

$$t < t\ |_{pos\_em} = \frac{\nu[1+(n-1)\beta]}{n+1+\nu+(n+\nu)(n-1)\beta}. \tag{35}$$

Combining the two conditions we have:

$$2 * t_m < t\mid_{private\_benefit} < t\mid_{pos\_em}. \tag{36}$$

That is, there is a range of emissions tax rates , complying with the above inequality, if $2 * t_m < t\mid_{pos\_em}$, or

$$\begin{aligned} & \frac{4\nu[1+(n-1)\beta]}{1+n(n+2)+2\{\nu+\beta(n-1)[(n(n+1)+\nu)]\}+(n-1)^2n^2\beta^2} \\ < & \frac{\nu[1+(n-1)\beta]}{n+1+\nu+(n+\nu)(n-1)\beta}, \end{aligned} \tag{37}$$

which simplifies into the following inequality:

$$3+2\nu-n(n-2)+2\beta(n-1)[\nu-n(n-1)]-(n-1)^2n^2\beta^2 < 0. \tag{38}$$

The above condition (38) can also be expressed as a condition on ν (technology efficiency):

$$\nu < \frac{[n-3+n\beta(n-1)][n+1+n\beta(n-1)]}{2+2(n-1)\beta}, \tag{39}$$

or as a condition on β (intensity of competition):

$$\beta > \frac{\nu-n(n-1)+\sqrt{(n+\nu)^2+3n^2}}{n^2(n-1)}. \tag{40}$$

Expression (39) simply states that, for a firm to benefit from a range of emissions tax rates, its technology efficiency in abating emissions must be high enough, implying a lower value of ν that satisfies equation (39). Note that for $n > 3$ and assuming Cournot competition ($\beta = 0$), if the emissions abatement technology is efficient enough ($\nu < \frac{1}{2}(n-3)(n+1)$), firms will always benefit from an emissions tax.

Furthermore, a firm will only benefit from an emissions tax if the intensity of competition is above a threshold set by the condition expressed in (40). If the emissions abatement technology is not efficient enough, the intensity of competition must be high and comply with (40) for a firm to benefit from an emissions tax.

Expression (39) can be rewritten a condition on n for the case of a n-firm Cournot setting ($\beta = 0$):

$$n > \sqrt{2(\nu+2)}+1. \tag{41}$$

For a duopoly ($n = 2$), condition (40) can be simplified as:

$$\beta > \frac{\nu-2+\sqrt{(2+\nu)^2+12}}{4}. \tag{42}$$

To study the interaction between β and n in relation to the threshold above which firms benefit from an emissions tax, let us define (from (38)) the following constraint function:

$$F(n,\beta) = 3 + 2\nu - n(n-2) + 2\beta(n-1)[\nu - n(n-1)] - (n-1)^2 n^2 \beta^2. \quad (43)$$

Applying the implicit function theorem:

$$\frac{\partial \beta}{\partial n} = -\frac{\partial F(n,\beta)/\partial n}{\partial F(n,\beta)/\partial \beta} = \frac{1 - n + \beta[\nu - 1 + n(4-3n)] + \beta^2[(3-2n)n - 1]}{(n-1)[n(n-1)(1+n\beta) - \nu]}. \quad (44)$$

Under the last two equations it is easy to prove that $\frac{\partial \beta}{\partial n} > 0$. In other words, the intensity of competition and the number of firms are substitutes for the condition under which a firm's profits increase with an increase in emissions tax, as illustrated in the following figure.

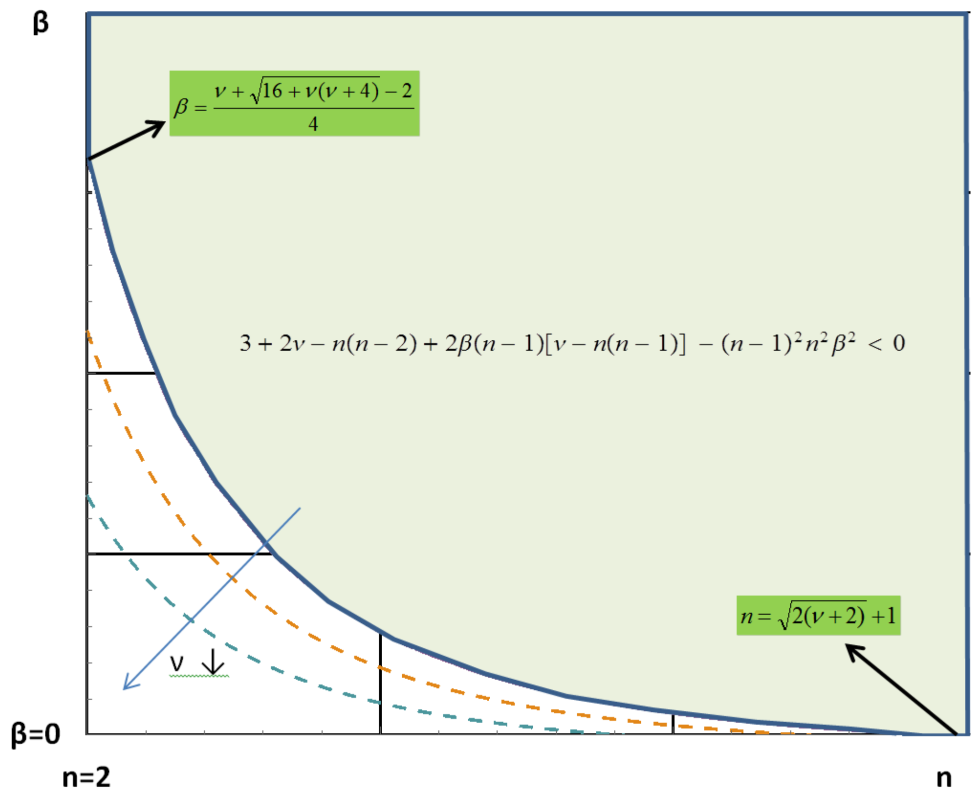

Figure 2: Intensity of competition (β) and number of firms (n)

The shaded area in Figure 2 represents the combination of intensity of competition and number of firms under which firms benefit from an emissions tax, i.e., that satisfy the condition set out in equation (38). The lower boundary of this area shifts down (lower boundary values for β and n) when the efficiency of emissions abatement technology increases (the value of ν decreases). Along the vertical axis we can find the threshold value of the intensity of competition for a duopoly, as per equation (42). Along the horizontal axis we can find the threshold value of the number of firms in a Cournot setting, as per equation (41). As the number of firms (n) and/or the intensity of competition (β) increase, the firms don't need to be as efficient in their emissions abatement activities, in order to benefit from an emissions tax.

As per our analytical framework, we can observe that a monopoly and a Cournot duopoly never benefit from an emissions tax.

Having shown that firms can actually benefit from an environmental tax policy under certain conditions, we now ask whether a benevolent government would ever choose an emissions tax rate that satisfies these conditions. To this end, in the next Section we determine the optimal (second best) emissions tax to find the conditions under which private and social interests align.

5.2 Tax on emissions and Social Welfare

The regulator sets its emissions tax to maximise the social welfare, taking into account the equilibrium quantities/prices and investment in emissions abatement that firms choose in order to maximise their profits.

A tax on emissions will reduce consumer surplus and environmental damage but will have an ambiguous impact on producer surplus.

Given the specific functional forms used, the equilibrium social welfare, equation (31), is quadratic and concave in relation to the emissions tax (t), as depicted in the following graph.

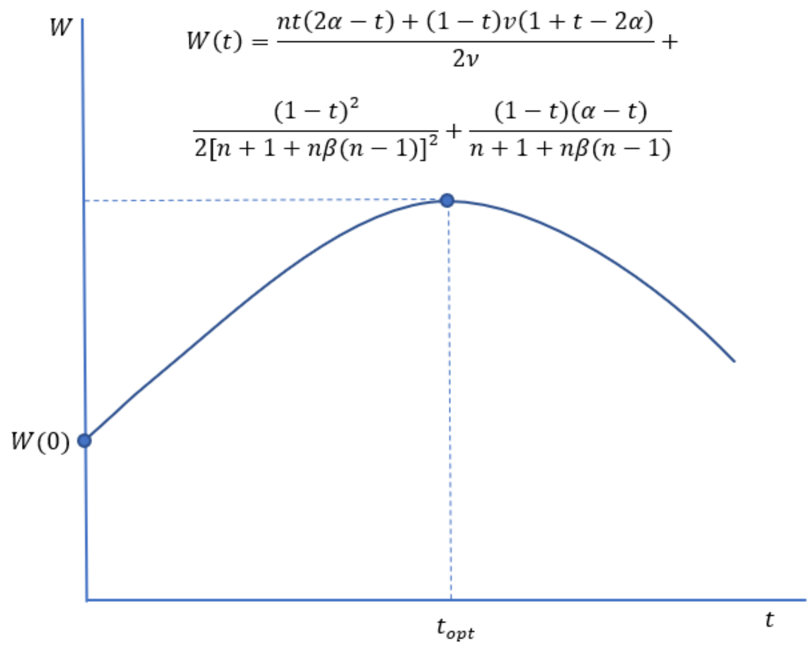

Figure 3: Welfare

The welfare function is maximised at the optimal emissions tax (second best)[8]:

$$t_{opt} = \frac{\alpha[n+1+n(n-1)\beta][n+1+\nu+(n+\nu)(n-1)\beta] - (a-c)\nu[1+(n-1)\beta]}{1+n[1+(n-1)\beta][n+2+\nu+(n+\nu)(n-1)\beta]}. \tag{45}$$

Remark: *From the linear supply schedules the Cournot duopoly game or corresponding market structure can be analysed by setting the competitiveness parameter $\beta = 0$. The Bertrand duopoly game or corresponding market structure can be analysed by setting the competitiveness parameter $\beta \to \infty$ .The optimal pollution or emissions tax in a Cournot setting is given by:*

$$t^C = \frac{\alpha(1+n)(1+n+v) - v(a-c)}{1+n(2+n+v)}. \tag{46}$$

The optimal pollution or emissions tax in a Bertrand setting is given by:

$$t^B = \alpha. \tag{47}$$

[8] Analysing the second order conditions, it is clear that this expression is indeed a maximum as $\frac{\partial^2 W(t_e)}{\partial t_e^2} = -\frac{n\{1+n[1+(n-1)\beta]\}[n+2+\nu+(n+\nu)(n-1)\beta]}{\nu[n+1+n(n-1)\beta]^2} < 0$.

Proposition 2, which determines the cost pass-through condition for the welfare to increase with an increase in emissions tax, can simply be stated as:

$$t < t_{opt}. \tag{48}$$

In the next Section we will analyse the conditions under which an emissions tax can be socially and privately beneficial.

5.3 Private and Social gains from an Emissions Tax

For firms to benefit from an optimal emissions tax, the optimal tax rate must be greater than the tax rate above which firms benefit from an emissions tax but under the positive emissions tax (CASE 3). The general condition can be set as

$$t\mid_{pos\_em} > t_{opt} > 2 * t_m, \tag{49}$$

where the three emissions tax thresholds are given by (45), (35) and (32).

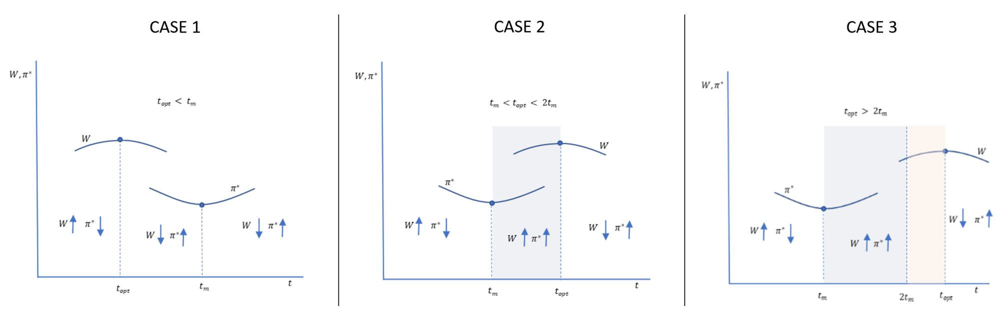

Figure 4: Private and Social outcomes from an Emissions Tax

Note that, under the established model conditions $t\mid_{pos\_em} > t_m$, always. However, $t\mid_{pos\_em} > t_{opt}$, results in the following condition:

$$\alpha < \frac{\nu}{n+\nu}\{1 - \frac{1}{[1+n+\nu+(n-1)(n+\nu)\beta]^2}\} \tag{50}$$

CASE 1 $(t_{opt} < t_m)$

$$\alpha < \frac{v[1+(n-1)\beta][3(n+1)+2v+(n-1)(3n+2v)\beta]}{k_1 k_2} \tag{51}$$

where

$$k_1 = 1 + n + \nu + (n-1)(n+\nu)\beta$$

and

$$k_2 = 1 + n(2+n) + 2v + 2(n-1)[n(n+1) + v]\beta + (n-1)^2 n^2 \beta^2$$

The limiting condition on α is smaller in the above expressions than in (50).

CASE 2 ($t_m < t_{opt} < 2t_m$)

$$\nu < \frac{[n - 3 + (n-1)n\beta][1 + n + (n-1)n\beta]}{2[1 + (n-1)\beta]} \tag{52}$$

and

$$\alpha > \frac{v[1 + (n-1)\beta][3(n+1) + 2v + (n-1)(3n+2v)\beta]}{k_1 k_2} \tag{53}$$

If $n \leqslant 3$, this additional condition applies: $n + n\beta(n-1) > 3$. In other words, CASE 2 will never occur under a Cournot duopoly.

CASE 3 ($t_{opt} > 2t_m$)

$$\nu < \frac{[n - 3 + (n-1)n\beta][1 + n + (n-1)n\beta]}{2[1 + (n-1)\beta]} \tag{54}$$

and

$$\alpha > \frac{v[5 + 5n(n+2) + 2v(2n+1) + k_3]}{k_1 k_2 [1 + n + (n-1)n\beta]} \tag{55}$$

where

$$k_3 = (n-1)(3n+1)[5(n+1)+4v]\beta + (n-1)^2[5n(2+3n)+2(1+6n)v]\beta^2 + (n-1)^3 n(5n+4v)\beta^3$$

If $n \leqslant 3$, this additional condition applies: $n + n\beta(n-1) > 3$. In other words, CASE 3 will never occur under a Cournot duopoly.

At the level of emissions tax where the firm's profit is equal to the profit with no emissions tax, for society to also benefit from an emissions tax, social welfare must also be increasing with an increase in emissions tax.

Under the positive emissions condition (38), the level of emissions tax at the firm's profit break-even point, where the firm's profit with an emissions tax is equal to its profit without the tax ($t = 2 * t_m$). The proposition below provides sufficient conditions for win-win outcomes.

Proposition 3: *Assuming that the positive emissions condition (38) is met and under an emissions tax, firms' profits are higher than under no tax and social welfare increases when emissions tax increases, if the following conditions are met:*

$$\alpha > \frac{\nu[1+(n-1)\beta]K_1}{K_2 K_3 K_4} \text{ and } \nu < \frac{K_2^2}{2+2(n-1)\beta}, \tag{56}$$

where

$$K_1 = 5+2\nu+n(10+5n+4\nu)+2\beta(n-1)[\nu+n(5+5n+4\nu)]+n\beta^2(n-1)^2(5n+4\nu);$$

$$K_2 = [1+n+n\beta(n-1)];$$

$$K_3 = [n+1+\nu+(n+\nu)(n-1)\beta];$$

and

$$K_4 = (n+1)^2 + 2v + 2(n-1)(n+n^2+v)\beta + (n-1)^2 n^2 \beta^2.$$

Proof: *The above condition on α results from substituting the emissions tax given by (34) into the inequality $\frac{dW}{dt} > 0$ and solving it for α. In order to ensure positive quantities, as per (39), under optimal emissions tax $\alpha < 1$, therefore:*

$$\frac{\nu[1+(n-1)\beta]K_1}{K_2 K_3 K_4} < 1 \tag{57}$$

or

$$\nu < \frac{K_2^2}{2+2(n-1)\beta}. \tag{58}$$

□

The above proposition simply states that, for firms to benefit from an emissions tax, whilst welfare increases with an increase in emissions tax, the marginal disutility of emissions (the cost that society allocates to environmental damage) must be high enough and above the threshold set by Proposition 4. The firm must also be efficient in undertaking emissions abatement (lower value of ν). When the above conditions are met, the social benefits from an increase in emissions tax are aligned with the private benefits.

Under a Bertrand setting (prices are strategic substitutes), these conditions are always met as the above conditions reduce to: $\alpha > 0$. In other words, in a price competition setting (Bertrand) with an emissions tax, firms' interests (profits) are always aligned with social interests as shown by Delbono and Lambertini (2022). Under a Cournot setting (quantities are strategic substitutes), the above conditions are simplified as follows:

$$\alpha > \frac{\nu[5(n+1)^2 + 2\nu(1+2n)]}{(n+1)(n+1+\nu)[(n+1)^2+2v]} \text{ and } \nu < \frac{(1+n)^2}{2}. \tag{59}$$

In the case of a Cournot duopoly, the positive emissions condition states that firms will never benefit from a tax on emissions.

6 Concluding Remarks

This paper investigates the conditions under which an emissions tax can lead to both social and private benefits. Considering a homogeneous product, symmetric n-firm oligopoly setting, we find that a welfare enhancing emissions tax can also benefit profit-maximising firms. Importantly, we do not assume that investment in emissions abatement leads to gains in productivity (or attracts a "green" premium in price).

Our key insight is that for an emissions tax to lead to an increase in profits, the emissions tax cost pass-through (the proportion of the emissions tax increase that is reflected in consumer prices) must be greater than the net emissions per unit of production, adjusted for the number of firms. As the intensity of competition increases, firms are more likely to benefit from an emissions tax, as it facilitates the exercise of market power. In the special cases of a monopoly or a Cournot duopoly, this condition is never met. This result may assist in understanding the political economy of emissions tax; more competitive industries are more likely to support the introduction of an emissions tax.

For the special case of linear demand function and linear emissions, we are also able to determine the sufficient conditions for social welfare and the private interests (firms' profits) to be aligned. For firms to benefit from an emissions tax, whilst welfare increases with an increase in emissions tax, the marginal disutility of emissions (the cost that society allocates to environmental damage) must be above a critical level determined by the extent of the market, the intensity of competition, the number of firms and the abatement technology efficiency. Under a Bertrand setting (prices are strategic substitutes), these conditions are always met, i.e., with an emissions tax, firms' interests (profits) are always aligned with social interests.

In our model, the investment in emissions reduction technology, which is undertaken as a response to the introduction of an emissions tax, is set independently of market competition. For example, in a Cournot duopoly, it turns out that the increase in price that follows as the emissions tax is passed through to consumers, is not enough to recover the net result of the investment outlay. As competition intensifies, the pass-through of the tax to consumers increases, to a point where the price rises more than offsets the net result of the investment outlay. The separation of the investment and quantity/price decisions implies that firms may earn positive profits under a Bertrand setting. The profits can be viewed as a tax credit.

Our analysis allows a better understanding of the trade-off between environmental policy, market competition and the so-called "win-win" outcome for firms and society.

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