Competition in supply functions and conjectural variations: a unified solution

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Abstract

This paper reconciles the concepts of *ex ante* and *ex post* supply function equilibria of, respectively, Klemperer and Meyer (1989) and Menezes and Quiggin (2020) with the conjectural variations equilibrium of Bowley (1924) and Bresnahan (1981). We show that under appropriate conditions, the *ex ante* and *ex post* equilibrium supply curves coincide with each other and with the consistent conjectural equilibrium solution. Further, this is a dominant strategy equilibrium for both *ex ante* and *ex post* games, and represents the case in which players are indifferent regarding move order. We explore the implications of our results for empirical work.

Keywords: Imperfect competition; games in supply functions; conjectural variations.

JEL Classification: D43; L13.

1 Introduction

Since the resurgence of game theory in the 1970s, industrial organization theory has been focused on the analysis of Nash equilibrium outcomes. The dominant solution concepts are based on the venerable Cournot (for the case of homogeneous products oligopoly) and Bertrand (for the case of monopolistic competition in differentiated products) solutions. Less commonly, but still broadly, used is the leader-follower model of Stackelberg. By contrast, much work in empirical industrial organization eschews game theoretic approaches in favour of the equally venerable conjectural variations approach, developed by Bowley (1924) and refined by Bresnahan's (1981) characterization of consistent conjectural equilibrium.

This situation is widely, and correctly, regarded as unsatisfactory. Reiss and Wolak (2017, p4326) observe

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> many studies estimate a continuous-valued parameter that they claim represents firm "conjectures" about how competitors will react in equilibrium. Currently there is no satisfactory economic interpretation of this [conjectural variation] parameter as a measure of firm behavior – save for firms in perfectly competitive, monopoly, Cournot–Nash and a few other special markets. We therefore see little or no value to drawing economic inferences about firm conduct from conjectural variation parameter estimate.

One alternative to Cournot and Bertrand games is to examine the case where the strategies are supply functions, that is, mappings from prices to quantities produced. Beginning with Grossman (1981) and Robson (1981), a number of writers (Klemperer and Meyer 1989, Vives 2011, Menezes and Quiggin 2012, and Delbono and Lambertini 2018) have adopted this approach.

The appeal of modelling competition in supply functions is that it allows for a continuum of solutions from competitive (Bertrand for the case of homogeneous goods) to Cournot. Thus, this approach has the potential to provide a theoretical basis for representing firm behaviour by a continuous valued parameter, thereby closing the gap between the game-theoretic literature on industrial organization and the econometric practice of the structural IO literature.

The main difficulty with modelling competition in supply functions is the multiplicity of equilibria. As shown by Klemperer and Meyer (1989), in the absence of restrictions on the set of supply functions allowed as strategies, any individually rational market outcome may be derived as a Nash equilibrium.

Two solutions to this problem have been put forward, which may be referred to as *ex ante* and *ex post*. Both solutions involve demand uncertainty, which is necessary to observe movements along the supply curve.

The *ex ante* solution, put forward by Klemperer and Meyer (see also Vives 2011), is based on the requirement that the same supply function be optimal for all values of a demand shock ε, assumed for simplicity to be additive. Provided ε has full support, Klemperer and Meyer demonstrate the existence of a unique solution. They derive an explicit solution for the linear case of affine demand and linear marginal cost.

The *ex post* solution, derived by Menezes and Quiggin (2012), begins by restricting firms' strategy sets to a single-parameter family of supply functions. The paradigmatic examples are strategy sets consisting of affine supply functions $S = \alpha + \beta p$ where α is the strategic choice variable, and β is an exogenous slope parameter ranging from 0 (Cournot) to ∞ (Bertrand).[1] For any given value of the demand shock ε, a unique solution is derived under standard conditions. The *ex post* solution is the natural choice when firms can observe demand shocks before choosing supply strategies. This approach is extended to the case of differentiated goods by Delbono and Lambertini (2018).

Menezes and Quiggin (2020) extend this single-valued solution by introducing the concept of the equilibrium strategic supply curve, which traces out the

[1] Vives (1986) shows how competition in linear supply schedules may arise in a two-stage model where the degree of flexibility is determined in the first stage

locus of equilibrium solutions as the demand shock ε varies over its range. In the *ex post* solution considered by Menezes and Quiggin, the equilibrium strategic supply curve will not, in general, be an element of the strategy space available to firms. For example, in the Cournot solution, the supply functions available to firms are vertical, but the equilibrium strategic supply curve is upward sloping.

The conjectural variations approach (Bowley 1924) was developed without a game-theoretic framework. The central idea is that in considering a change in output, a firm conjectures that other firms will respond by changing their own outputs. The firm chooses output to maximize profit conditional on this conjectured reaction. The equilibrium outcome will not, in general, be consistent with the firm's conjectures. For example, the equilibrium with Cournot conjectures will not coincide with the Cournot-Nash solution. Bresnahan (1981) develops the idea of consistent conjectural equilibrium, in which equilibrium outcomes are consistent with the firm's conjectures.

The problem of inconsistent conjectures appears analogous to the distinction between supply functions, considered as elements of the strategy set available to firms, and strategic supply curves, considered as loci of *ex post* equilibrium outcomes. In this paper, we explain the relationship between *ex ante*, *ex post* and conjectural variations approaches.

We first develop the crucial distinction between supply functions and strategic supply curves, outlined above. Next, we show that the conjectural variations concept can be given a game-theoretic foundation. More precisely we show that the conjectural variations solution of Bowley (1924) and Bresnahan (1981) may be interpreted as the *ex post* equilibrium of a game with linear supply functions as strategies. This is the unique non-cooperative Nash equilibrium for the game in question. This result applies whether or not conjectures are consistent in the sense of Bresnahan (1981).

We next prove the main result of the paper. Under appropriate conditions, the *ex ante* and *ex post* equilibrium supply curves coincide with each other and with the consistent conjectural equilibrium solution. Further, this is a dominant strategy equilibrium for both *ex ante* and *ex post* games.

We focus throughout on the linear case, for which closed-form solutions can be derived using all three equilibrium concepts. These results are derived for the homogeneous products case.

Next, following Delbono and Lambertini (2018), we show that this solution represents the case in which players in a Stackelberg game are indifferent regarding the allocation of leader and follower roles. Finally, we discuss some implications for empirical work.

2 Model

As in Klemperer and Meyer (1989), we focus on the case of symmetric duopoly. There are two firms denoted i and $j \neq i$. The output of firm i is denoted q_i. Industry output is $Q = q_i + q_j$. We define the cost function $C(\cdot)$, common to both firms.

We will focus on the case of quadratic costs

$$C(q) = cq^2/2$$

that is, linear marginal costs

$$C'(q) = cq.$$

We assume that consumers do not behave strategically, so that the demand curve may be taken as exogenously given. We consider the case where the demand curve is linear and subject to additive shocks, that is

$$D(p, \varepsilon) = a - bp + \varepsilon \tag{1}$$

where p denotes the market price and the support of ε is a connected subset of $\mathbb{R}$. The inverse demand is

$$P(Q, \varepsilon) = \frac{a + \varepsilon - Q}{b}$$

Given outputs q_i, q_j and a demand shock ε, profit for firm i is given by

$$\pi_i(q_i : \varepsilon, q_j) = P(q_i + q_j, \varepsilon)q_i - C(q_i).$$

$$= \frac{a + \varepsilon - q_i - q_j}{b} q_i - cq_i^2/2$$

Most of the qualitative results derived below extend to the general case of convex costs and log-concave demand. However, unique closed-form solutions are currently available only for the linear case modelled here.

3 *Ex ante* and *ex post* equilibria

In this section, we draw a distinction between supply functions (strategies available to firms) and supply curves (loci of market outcomes as ε varies). We show how this distinction helps to characterise *ex ante* and *ex post* equilibria.

A supply function is a strategy available to firms in an oligopoly game. In a Nash equilibrium, this is the behavior imputed to firms by their opponents, when considering the profitability of deviation. It may or may not correspond to the way firms conceive of their own choices.

A supply curve is a (strategic) equilibrium outcome for firms. As usual, firm-level supply curves may be aggregated to yield a strategic industry supply curve.

3.1 Supply functions

A supply function for firm i is a mapping $S_i : [0, \infty) \to [0, \infty)$, where $S_i(p)$ is the output for firm i associated with price p. In both *ex ante* and *ex post* solutions, the set of strategies available to firms consists of supply functions. We define the class of supply functions $\mathcal{S}$ to consist of all differentiable functions

$S : [0, \infty) \to [0, \infty)$, with $S_p > 0$. A *game of competition in supply functions* is a non-cooperative game where the strategy sets available to players are subsets of $\mathcal{S}$.

Klemperer and Meyer consider games where the strategy set is $\mathcal{S}$ and the value function for firm i is *ex ante* expected profit

$$E[\pi_i] = E_\varepsilon [P(S_i(p) + S_j(p), \varepsilon) S_i - C(S_i(p))].$$

subject to the market-clearing condition

$$P(S_i(p) + S_j(p)) = p.$$

Klemperer and Meyer show that, when the strategy space is $\mathcal{S}$, equilibrium requires maximization of π_i (and π_j) for every ε.

In the *ex post* problem, the value of ε is observed first, followed by a game of competition in supply functions where the value function is *ex post* profit, conditional on ε.

$$\pi_i(S_i, S_j, \varepsilon) = P(S_i(p) + S_j(p), \varepsilon) S_i(p) - C(S_i(p)).$$

For the *ex post* case, Menezes and Quiggin (2012, 2020) consider strategy spaces of the form

$$S_i(p) = \alpha_i + \beta p,$$

where α_i is the strategic variable for firm i and β is an exogenous parameter representing the competitiveness of the market. Menezes and Quiggin prove the existence of a unique solution for this case.

It is *crucial* to observe that the strategic variable α_i need not correspond to the way firm i models their own choices. Firm i may consider itself as setting a price, quantity, markup or some other variable. Rather α_i reflects the beliefs of firm $j \neq i$ about the strategy being pursued by i. When considering a possible deviation from its own strategy, which may result in a change in the price p, firm j evaluates the consequences on the assumptions that firm i will hold their strategic choice α_i constant, and will produce $S(p, \alpha_i; \beta) = \alpha_i + \beta p$. This is the standard Nash equilibrium concept.[2]

3.2 Equilibrium supply curves

We now consider equilibrium supply curves in games of competition in supply functions, using the *ex ante* and *ex post* equilibrium concepts. As a baseline, we first restate the solution for the competitive case.

In the standard Marshallian analysis, the supply curve is a locus of pairs (p, Q) where $Q = q_1 + q_2$ is the aggregate output of firms in the industry at

[2]This point has been made by Klemperer and Meyer (1989) and subsequent writers. Nevertheless, in economic applications of game theory, where the choice of strategy space is in question, it is common to see discussion of the way in which players conceive of their own choices, rather than the way they represent the choices of their opponents.

price p. The supply curve coincides with the locus of market equilibria traced out as ε varies over its support.

The same representation may be extended to the locus of (Nash) equilibrium outcomes in non-competitive markets, represented as games of competition in supply functions. As in the competitive case, the strategic industry supply curve is the locus of market equilibria $(Q^* (\varepsilon), p^* (\varepsilon))$ as ε ranges over its support.

3.3 *Ex ante* equilibrium supply curve

In the *ex ante* solution, the supply curve for each firm is simply the firm's equilibrium supply function, that is, $q(p) = S(p)$. The strategic industry supply curve is the sum of the firms' equilibrium supply functions, that is, $Q(p) = 2S(p)$.

As shown by Klemperer and Meyer, the symmetric *ex ante* equilibrium supply curve must satisfy

$$q'(p) = S'(p) = \frac{S(p)}{p - C'(S(p)) + D_p(p)}. \tag{2}$$

For the linear case, this becomes

$$q'(p) = \frac{S(p)}{p - cq - b}.$$

Klemperer and Meyer show that all supply functions satisfying the *ex ante* condition (2) are inadmissible as equilibria for the linear case, with the exception of the linear supply function with slope

$$\hat{\beta} = \frac{1}{2}\left(-b + \sqrt{b^2 + \frac{4b}{c}}\right).$$

Hence, this is the unique *ex ante* equilibrium supply curve.

3.4 *Ex post* equilibrium supply curve

In the *ex post* solution, the supply curve is a locus of equilibria, one for each value of ε. It does not, in general, coincide with an element of the strategy space available to firms. For the case where the strategy space consists of affine supply functions $S(p, \alpha_j; \beta) = \alpha_j + \beta p$, Menezes and Quiggin (2020) derive the residual demand for firm i as

$$D_i(P, \varepsilon, \alpha_j) = D(P, \varepsilon) - \beta P - \alpha_j \tag{3}$$

which yields, for the linear case

$$D_i(P, \varepsilon, \alpha_j) = a + \varepsilon - \alpha_j - (\beta + b)P \tag{4}$$

Taking the first-order conditions for each firm, and imposing market clearing yields the symmetric equilibrium firm supply curve

$$q(p) = (p - C'(q))(\beta - D_p(p, \varepsilon(p))) \tag{5}$$

or in the linear case

$$q(p) = (p - cq)(\beta + b).$$

Rearranging, we obtain

$$q(p) = \frac{(b + \beta)}{1 + c(b + \beta)} p.$$

The slope of the firm's strategic supply curve is

$$q'(p) = \frac{(b + \beta)}{1 + c(b + \beta)}.$$

The slope $q'(p)$ will not, in general, be equal to β, the slope of the supply functions in the strategy space. As will be shown below, the two coincide precisely when the *ex ante* and *ex post* solutions coincide.

4 Conjectural variations

As noted by Wolak and Reiss (2017), the conjectural variations approach remains dominant in applied IO. Nevertheless, this approach has been almost universally dismissed by IO theorists.

The main objections are summarised by Escrihuela-Villar (2015)

> This model has often been criticized since ad hoc conjectural variations are generally inconsistent with rational behavior except at the equilibrium point (see, among others, Makowski 1987). It has also been argued that its solution is not entirely satisfactory from a game-theoretic point of view because it describes "dynamics" based on a static model. Basically, in the theoretical literature, conjectural variations have been criticized for their lack of theoretical foundations, at least in static models (for additional references and a discussion, see Martin (2002, 50–51)).

In this section, we show that the objections cited by Escrihuela-Villar may be overcome. Any conjectural variations equilibrium may be represented as the Nash equilibrium of an *ex post* game in linear supply functions.

The conjectural variations parameter is derived as a transformation of the slope of the supply functions available as strategies in the corresponding *ex post* game. The conjectural variations parameter can be interpreted as representing the competitiveness of the market environment faced by each firm.

4.1 Conjectural variations equilibrium

In the conjectural variations equilibrium concept, firm i chooses output q_i on the assumption that firm j will respond by changing output by θq_i. Hence firm i acts as a monopolist facing a residual inverse demand curve with slope

$$\frac{\partial P}{\partial q_i} = -\frac{1+\theta}{b}$$

and similarly for firm j. The first-order conditions on q

$$P + q\frac{\partial P}{\partial q} = cq$$

that is,

$$P - \frac{1+\theta}{b}q - cq = 0 \tag{6}$$

along with market-clearing,

$$P(Q,\varepsilon) = \frac{a+\varepsilon-2q}{b} \tag{7}$$

determine the unique symmetric equilibrium.

Combining (6) and (7) yields the conjectural equilibrium:

$$q = \frac{a+\varepsilon}{3+\theta+bc}$$
$$p = \frac{a+\varepsilon}{b}\frac{(1+\theta+bc)}{(3+\theta+bc)}$$

so that

$$q(p) = \frac{b}{(1+\theta+cb)}p$$

Allowing ε to vary over its full range, we obtain the conjectural variations strategic supply curve.

4.2 Conjectural variations and supply functions

We now show that, for the linear case, the conjectural variations solution may be derived as the Nash equilibrium of a game with linear supply functions as strategies.

Proposition 1 *Fix ε and let demand be given by $D = a+\varepsilon-bp$. Suppose that the conjectural variation parameter is given by θ. Then the conjectured behavior is equivalent that of a firm with strategy*

$$q = \alpha + \beta p$$

where

$$\beta = \frac{-\theta}{1+\theta} b$$

Proof. Consider the game in linear supply strategies. The residual inverse demand faced by firm i for given ε is

$$\begin{aligned} P(q_i, \varepsilon) &= \frac{a + \varepsilon - Q}{b} \\ &= \frac{a + \varepsilon - q_i - (\alpha_j + \beta p)}{b} \end{aligned}$$

which yields

$$P(q_i, \varepsilon) = \frac{a + \varepsilon - q_i - \alpha_j}{b + \beta}$$

This implies

$$\frac{\partial P}{\partial q_i} = \frac{-1}{b + \beta}$$

Hence

$$\frac{\partial q_j}{\partial q_i} = \frac{-\beta}{b + \beta} \equiv \theta$$

Conversely

$$\beta = \frac{-\theta}{1+\theta} b \tag{8}$$

■

If (8) holds, the first-order condition facing i in the conjectural variations problem with parameter θ will be the same as the corresponding condition in the *ex post* game of linear supply functions, and similarly for j. Hence, the conjectural variations equilibrium will coincide with the Nash equilibrium for the *ex post* game of linear supply functions.

Using Proposition 1 to convert θ to β we see that the conjectural equilibrium solution coincides with the *ex post* solution

$$\begin{aligned} q(p) &= \frac{b}{\left(1 + \frac{-\beta}{b+\beta} + cb\right)} p \\ &= \frac{(b+\beta)}{1 + c(b+\beta)} p \end{aligned}$$

In general, the *ex post* solution will not satisfy $q'(p) = \beta$. Interpreted in terms of conjectural variations, and applying Proposition 1, this is a restatement of the observation that conjectures are not, in general, consistent. Conversely, as we will show below, the consistent conjectures equilibrium of Bresnahan (1981) corresponds to the case when $q'(p) = \beta$.

5 When do *ex ante*, *ex post* and conjectural variations equilibria coincide?

We now consider the relationship between the equilibrium supply curves for the three concepts discussed above. We first derive a general characterization of the relationship between *ex ante* and *ex post* equilibrium concepts. This characterization may be made explicit for the case of linear demand and linear marginal cost (quadratic total cost), for which a unique *ex ante* solution is known to exist.

Consider the *ex post* game with strategies $S = \alpha + \beta p$. In terms of the residual demand facing i, an increase in α_j is just like a reduction in ε. If, for some $\hat{\beta}$, the *ex post* equilibrium is $\alpha = 0$ for all values of ε, then $\alpha_i = 0$ is a best reply to all possible strategic choices $S_j = \alpha_j + \hat{\beta} p$ for j and vice versa. Equivalently, for all values of ε, $S = \hat{\beta} p$ is a dominant strategy equilibrium for the *ex post* game. It follows that, in the *ex ante* game, $S = \hat{\beta} p$ is the best reply to itself, and therefore defines a symmetric Klemperer-Meyer equilibrium.

We now show explicitly that the value of $\hat{\beta}$ for which $\alpha = 0$ is a dominant strategy equilibrium in the *ex post* game coincides with the value of $\hat{\beta}$ derived by Klemperer and Meyer as an *ex ante* equilibrium solution for the case of linear demand and marginal cost.

The necessary condition for an *ex ante* equilibrium,

$$q'(p) = \frac{q}{p - cS} - b,$$

may be rewritten as

$$-b = q'(p) - \frac{q}{p - cq}. \tag{9}$$

Equation (9) will be satisfied by a linear supply function $S(p) = \hat{\beta} p$ if and only if

$$-b = \hat{\beta} - \frac{\hat{\beta} p}{p - c\hat{\beta} p}. \tag{10}$$

Since $0 \leq q'(p) = \hat{\beta} < \infty$ the function $S(p) = \hat{\beta} p$ is an *ex ante* equilibrium if and only if (10) holds

Now consider the *ex post* problem with strategies $S(p) = \alpha + \hat{\beta} p$. The *ex post* solution (5) yields

$$\begin{aligned} q(p) &= (p - cq)\left(\hat{\beta} + b\right) \\ &= (p - cq)\left(\hat{\beta} - \left(\hat{\beta} - \frac{\hat{\beta} p}{p - cq}\right)\right) \\ &= \hat{\beta} p. \end{aligned}$$

That is, the *ex post* equilibrium solution is equal to $\hat{\beta} p$ if and only if (10) holds, that is, if and only if the *ex ante* equilibrium solution is equal to $\hat{\beta} p$.

As noted above, Klemperer and Meyer derive a closed form solution for $\hat{\beta}$, using the differential equation (2). The analysis above yields an elementary derivation for $\hat{\beta}$. The demand condition (10) becomes

$$-b = \hat{\beta} - \frac{\hat{\beta} p}{p - c\hat{\beta} p},$$

or

$$-b = \hat{\beta} - \frac{\hat{\beta}}{1 - c\hat{\beta}}$$

which may be rearranged to yield

$$\hat{\beta}^2 + \hat{\beta} b - \frac{b}{c} = 0, \tag{11}$$

which is true if and only if

$$\hat{\beta} = \frac{1}{2}\left(-b + \sqrt{b^2 + \frac{4b}{c}}\right). \tag{12}$$

as derived by Klemperer and Meyer.

Conversely, if

$$\alpha + \beta p = \frac{(b + \beta)}{(1 + c\,(b + \beta))} p,$$

the solution derived above shows that

$$\alpha = -\frac{\left(c\beta^2 + c\beta b - b\right)}{(1 + c\,[\beta + b])} p, \tag{13}$$

so that $\alpha = 0$ if and only if $\left(c\beta^2 + c\beta b - b\right) = 0$, that is, only if $\beta = \hat{\beta}$.

5.1 Consistent conjectural variations

We now show that, for any ε, the solution for $\hat{\beta}$ derived immediately above, coincides with the Bresnahan Consistent Conjectural Equilibrium interpreted, as in Section 4, as the solution to an ex post game in linear supply functions.

Recall (8)

$$\beta = \frac{-\theta b}{1 + \theta}.$$

Hence, (11)

$$\hat{\beta}^2 + \hat{\beta} b - \frac{b}{c} = 0,$$

yields

$$\left(\frac{\theta b}{\theta+1}\right)^{2}-\left(\frac{\theta b^{2}}{\theta+1}\right)-\frac{b}{c}=0$$

which after some manipulation yields

$$\theta=\frac{-(cb+2)+\sqrt{c^{2}b^{2}+4cb}}{2}$$

which is the solution derived by Bresnahan (1981) (with the change of notation $b=1/d$)

6 Endogenous Move Order

Thus far, we have considered the case of a normal-form simultaneous-move game. This analysis leaves open the possibility that firms may benefit by pre-committing to a particular strategy, for example by announcing their supply function. The classic case is that of competition in quantities, where the first mover in a Stackelberg equilibrium earns higher profits than they would in a Cournot-Nash equilibrium. Dowrick (1986) and Hamilton and Slutsky (1990) analyse extended duopoly games in which firms restricted to Cournot supply functions ($\beta=0$) can set their quantities in one of two possible periods.

Delbono and Lambertini (2018) examine this question in relation to the *ex post* model with linear supply functions. They derive conditions under which firms will be indifferent as to move order.

In this section, we extend these results, and use the analysis above to clarify the nature of the equilibrium involved. We first show that the case where *ex ante*, *ex post* and consistent conjectural variations solutions coincide is a unique dominant strategy equilibrium. In such an equilibrium, players are indifferent as to move order (Hamilton and Slutsky 1993).

This is shown in:

Proposition 2 *Under the stated conditions,*
(i) $S(p)=\hat{\beta}p$ *is the unique ex ante equilibrium;*
(ii) $\alpha_i=\alpha_j=0$ *is a dominant strategy equilibrium for the ex post problem with strategies* $S(p)=\alpha+\hat{\beta}p$; *and*
(iii) This is the consistent conjectures equilibrium.

Proof. If

$$D_p(p)=\hat{\beta}-\frac{\hat{\beta}p}{p-C'\left(\hat{\beta}p\right)},$$

then the *ex ante* condition

$$S'(p)=\frac{S}{p-C'(S)}+D_p(p)$$

becomes

$$S'(p) = \hat{\beta} + \frac{S}{p - C'(S)} - \frac{\hat{\beta} p}{p - C'\left(\hat{\beta} p\right)}.$$

Consider a supply function $\tilde{S}$ satisfying the *ex ante* condition (2). Suppose for some p, $\tilde{S}(p) > (<,=)\hat{\beta}p$. Let $\tilde{\beta}(p) = \tilde{S}(p)/p$. Then

$$\begin{aligned} \tilde{S}'(p) &= \hat{\beta} + \frac{\tilde{\beta}(p)\,p}{p - C'\left(\tilde{\beta}(p)\,p\right)} - \frac{\hat{\beta} p}{p - C'\left(\hat{\beta} p\right)} \\ &> (<,=)\,\hat{\beta} + \frac{\tilde{\beta}(p)\,p}{p - C'\left(\hat{\beta} p\right)} - \frac{\hat{\beta} p}{p - C'\left(\hat{\beta} p\right)} \\ &= \hat{\beta} + \left(\tilde{\beta}(p) - \hat{\beta}\right) \frac{p}{p - C'\left(\hat{\beta} p\right)} \\ &> (<,=)\,\tilde{\beta}(p), \end{aligned}$$

where the first inequality follows from convexity of C, and the second from the fact that $\frac{p}{p-C'\left(\hat{\beta}p\right)} > 1$. Hence, if $\tilde{S}(p) \neq \hat{\beta}p$ is a solution to (2), $\tilde{S}'(p)$ eventually goes to ∞ or 0. Hence $S(p) = \hat{\beta}p$ is the unique solution.

For the dominance result, recall that, as regards the residual demand facing firm i, a change in ε has the same effect as an equal and opposite change in α_j. Hence, if $\alpha_i = 0$ is a best reply to $\alpha_j = 0$ for all ε it is a best reply to any α_j for all ε ■

Now consider general values of $\beta \in [0, \infty)$. First we may observe that the *ex post* solution (5) is increasing in β, and also in $b = -D_p$. It follows that the equilibrium price, for given ε, is decreasing in β and b.

Inspection of (13) shows α is linear in p and is positive (negative) when $\beta < (>)\hat{\beta}$. Further, since p is linear in ε, the same relationship holds between α and ε. Hence, when α is positive and increasing (negative and decreasing) in p, the slope of the equilibrium supply curve is greater than (less than β). In both cases, as shown above, equality holds for $\beta = \hat{\beta}$.

As noted above, in terms of the residual demand facing i, an increase in α_j is just like a reduction in ε. It follows that strategies α_i, α_j are complements, (substitutes, neutral) whenever $\beta < (>,=)\hat{\beta}$.

We record these points as a proposition.

Proposition 3 *Assume linear demand and quadratic costs, and let $\hat{\beta}$ be the slope of the supply curve in the ex ante solution. Then, for the ex post problem*
(i) If $\beta < (>)\hat{\beta}$, then for all ε, the market equilibrium price in the ex post solution is greater (less) and the equilibrium quantity less (greater) than for the market equilibrium in the ex ante solution
(ii) If $\beta < (>)\hat{\beta}$, then $q'(p) > (<)\ \beta$ where $q'(p)$ is the slope of the equilibrium supply curve

(iii) Strategies α_i, α_j *are complements, (substitutes, neutral) whenever* $\beta < (>, =)\ \hat{\beta}$.

Prop 3(iii) may be interpreted in terms of endogenous move order. The standard analysis of the Stackelberg game applies when $\beta = 0$. In this case, strategies are substitutes, and players will prefer to move first. If move order is endogenous in this case, and neither player has an advantage in timing, the unique equilibrium is that of simultaneous moves. On the other hand, for $\beta \to \infty$, strategies are complements and players prefer to move second.

Delbono and Lambertini (2018) examine the problem of endogenous move order with competition in linear supply functions. They consider the case of differentiated products, constant marginal costs and non-stochastic demand, derived from the utility function

$$U = \alpha(q_i + q_j) - \frac{1}{2}\left((q_i)^2 + (q_j)^2 + 2\sigma q_i q_j\right)$$

where σ is an inverse measure of production differentiation.

As in Menezes and Quiggin (2012), strategies are of the form $S = \alpha + \beta p$, where the strategic variable is α, the intercept of the supply function, while β. the slope of the supply function, is determined exogenously.

Delbono and Lambertini show that, as in the homogenous good game analyzed here, strategies are substitutes (complements) for $\beta < (>)\ \frac{1}{\sqrt{1-\sigma^2}}$. In a game with observable delay, the case $\beta < \frac{1}{\sqrt{1-\sigma^2}}$ produces a unique equilibrium in which both players move as soon as possible. The case $\beta > \frac{1}{\sqrt{1-\sigma^2}}$ produces two pure-strategy Nash equilibria, in each of which one player moves first and the other moves second. A similar analysis applies to the linear case modelled here.

The case analyzed by Delbono and Lambertini coincides with the linear case modelled above for homogenous goods and zero costs, that is $\sigma = 1, c = 0$. In this case, equation (12) shows $\hat{\beta} = \infty$, that is, strategies are substitutes for all $\beta \in [0, \infty)$. Hence the unique equilibrium involves simultaneous play. The higher is c, that is, the more convex is the cost function, the lower is $\hat{\beta}$ and the wider the range of β for which strategies are complements.

7 Empirical application

Menezes and Quiggin (2020) discuss empirical estimation in the context of the *ex post* strategic supply curve, and observe that analysis using the conjectural variations approach may be reinterpreted to yield estimates of β. The results here show that this interpretation may be undertaken in reverse for the *ex post* strategic supply curve. In view of the results derived above, estimates of the *ex post* strategic supply curve may be regarded, in terms of conjectural variations, as incorporating the maintained hypothesis of consistent conjectural variations.

With this interpretation, the simulations of Green and Newbery (1992) may be regarded as representing the predictions of a consistent conjectural equilibrium model. The empirical analysis of Wolfram (1999), which finds markups lower than those simulated by Green and Newbery, represents a rejection of the constraint imposed by the maintained hypothesis of consistent conjectural variations. As suggested by Wolfram (1999), this outcome might reflect limit pricing due to the threat of entry.

Menezes and Quiggin (2020) also provide a theoretical basis for the analysis of cost pass-through, consistent with the work of Weyl and Fabinger (2013). Using the results in the present paper, this analysis may be restated in terms of a game-theoretic interpretation of conjectural variations equilibrium. This suggests the possibility of integrating the largely atheoretical literature on cost pass-through with the structural IO literature based on conjectural variations models.

8 Conclusion

The disconnect between theoretical and empirical approaches to the modelling of oligopolistic markets is longstanding and problematic. In this paper, we have shown how the conjectural variations approach, widely used in empirical practice, can be given a game-theoretic representation in terms of competition in supply functions. Further, we have shown that the consistent conjectures equilibrium corresponds to the case where ex ante and ex post models of competition in supply functions yield the same equilibrium industry supply curve.

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